WARPED SPACE: A GEOGRAPHY OF DISTANCE DECAY*

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Distance decay is one of geography’s core concepts, yet its own spatial properties have been largely neglected. We illustrate the use of the expansion method in assessing the spatial variation of distance-decay parameters within the general context of gravity models. The approach enables the portrayal of “space warping,” said to occur when equivalent distances have spatially uneven effects on interaction. The concepts are examined empirically through the estimation of a competing destination model for out-migration from five State Economic Areas over the 1965–70 period. Four of the five origins are found to have spatially unstable frictions of distance. We conclude by suggesting that distance decay is a contextual, rather than a universal, effect. Key Words: distance decay, spatial interaction models, gravity model, competing destination model, migration, expansion method.

Few concepts are more central to the discipline of geography than distance decay. Distance plays a role in the distribution of ideas, technology, population, and interaction of various types, underpins a host of empirical regularities, and constitutes the basis for Tobler’s (1970) “first law of geography.” The formalization of distance decay as a measurable entity can be traced to the “social physics” school of Stewart, Warnitz, and Zipf, whose work laid the foundation for a host of studies on spatial interaction by geographers in the 1960s and beyond. The gravity model was by far the most important tool in defining the concept. Its friction of distance parameter became a precise analog for distance decay, while the parallel development of modern computing technology enabled estimation of the effect of distance on an immense variety of phenomena.

The friction of distance has been so consistently observed that most research is not concerned with whether it occurs, but with the comparison of estimated values across different types of contexts, that is, interaction types, settings, and time. Yet most research designs implicitly assume that the friction of distance has—within any single model—an unvarying effect across the spatial units from which data are derived and estimates made. This assumption remains a curious oversight in what is an otherwise fundamentally spatial concept.

If the friction of distance for any given origin varies over space, then equivalent distances in different locations result in varied effects on interaction. Such variation can be conceptualized as a distortion of physical space, since the property of equivalent distance effects across space no longer holds. This leads us to suggest that spatial variation in the friction of distance may be cartographically represented as a transformation, or “warping,” of the space.
within which the interaction takes place. This paper identifies spatial variation in distance-decay parameters and portrays its consequences.

**Towards a Geography of the Distance-Decay Parameter**

To begin, consider the common origin-based gravity model, which assumes that spatial interaction is a function of two general factors: the potential of the destination to attract flows and the degree to which the origin and destination are separated. A common formulation is:

\[ I_j = kM_i^a d_i^b \]  

(1)

where \( I_j \) is the interaction between the origin in question and destination \( j \), \( M_i \) is the mass (or attractiveness) of destination \( j \), and \( d_i \) is the distance between the origin and destination \( j \). Under an assumption of asymptotic normality, the model's parameters, \( k \), \( a \), and \( b \), may be empirically estimated using ordinary least squares after logarithmic transformation:

\[ \ln(I_j) = \ln(k) + a\ln(M_i) + b\ln(d_i) \]  

(2)

The friction of distance parameter \( b \) describes the rate at which interaction declines with distance. Highly negative values indicate that interaction is locally confined, whereas less negative values are associated with a broader pattern of spatial interaction. As noted above, variation in this coefficient has been the subject of considerable investigation across a host of research contexts (for reviews see Haynes and Fotheringham 1984; Fotheringham and O'Kelly 1989; Olsson 1965). Authors have examined the empirical occurrence of distance decay for different types of interaction (e.g., Alcaly 1967; O'Sullivan 1973; Murdie 1965), in different time periods (e.g., Taaffe 1962), and across various origins (e.g., Hägerstrand 1957; Leinbach 1973; Greenwood and Sweetland 1972; White 1985).

Thus, a considerable body of empirical work points to the sensitivity of friction of distance parameters across model contexts. Yet within any particular modeling enterprise the assumption of spatial parameter stability is usually implicit and unexamined. In other words, for movement out of any one origin, the parameter has been assumed to be stable across the range of destinations toward which flows are directed. This is a heroic assumption, however, since it suggests that the effect of distance out of a particular origin is unmodified in spite of the diversity of places within the spatial system. Where the assumption is not made, one may examine the possible existence of place-to-place differences in the relationship between distance and interaction. Specifically, distance in one area may have one effect on interaction, while the same distance in another area may have a different effect. The typical interaction model, such as the gravity model, ignores these biases and results in the same predicted interaction for all observations with the same distance, *ceteris paribus*.

To place in focus variation in the friction of distance, consider the regularity with which the aforementioned studies identified across-model variation in the distance-decay parameter. Authors found that parameters could vary over type of interaction, time, or setting; this variation was thought to reflect, for example, changing technology, differences in the potential for movement among different phenomena, and differences between various groups' propensity for interaction. A similar interpretation can apply to spatial parameter variation within particular models, with such variation being suggestive of geographic differences in transport technology or network accessibility, regional development inequalities over space that facilitate or hinder interaction, or destination-dependent differences in the perception of distance held by interactors in the origin.

Two-dimensional spatial variation in parameters can be identified using a variant of the expansion method (Casetti 1972, 1986) known as trend surface expansions (Casetti and Jones 1987; Jones 1983, 1984), or TSEs. TSEs involve redefining the parameters of an initial model in terms of the Cartesian coordinates of the obser-
vations. An initial model may be represented by the linearized version of the gravity model (2). The distance-decay parameter b may be redefined as a function of the spatial coordinates (x, y) of the destinations, yielding an expansion equation. For example, b may be redefined in terms of a second-order polynomial of these coordinates:

\[ b = b_0 + b_1x + b_2y + b_3xy + b_4x^2 + b_5y^2 \]  

(3)

Substituting the right hand side of equation (3) into the linearized version of the gravity model produces a terminal model:

\[ \ln(I_j) = \ln(k) + a\ln(M_j) + b_0\ln(d_{ij}) + b_1x\ln(d_{ij}) + b_2y\ln(d_{ij}) + b_3xy\ln(d_{ij}) + b_4x^2\ln(d_{ij}) + b_5y^2\ln(d_{ij}) \]  

(4)

Equation (4) may be estimated using ordinary least squares regression. The estimated values of parameters b_0 through b_5 can be substituted into equation (3) to yield an estimated expansion equation, revealing the two-dimensional variation of b. This variation can then be mapped as a set of contours representing equal distance friction. The resolution of this map will depend on the order of the polynomial employed in equation (3) and the magnitudes and signs of the coefficients b_0 through b_5.

The contoured parameter values b = f(x, y) represent the spatially varying effect that distance has on whatever form of interaction is being modeled. Smaller magnitudes imply that interaction to destinations involves limited friction, while the opposite holds for destinations with large values.

An alternative method of illustrating the spatially varying effect of distance is “space warping” (Plane 1984; also see Huff and Jenks 1968). Plane (1984) rearranged a basic gravity model, using distance as the dependent variable, and inferred spatial distortion based on the ratio of predicted to actual distances between state centroids. Our methodology, in contrast, directly addresses spatial variation in b by redefining the parameter in terms of the spatial coordinates of the observations. This procedure allows us not only to identify the uneven effects of distance, but to assess their statistical significance. These effects can be portrayed as a stretched or contracted space. When b values are large, the warped space extends away from the origin, indicating greater friction of distance; when b values are relatively small, the space is compressed, indicating less friction of distance.

To illustrate, consider that in a conventional study in which the estimated distance-decay effect is constant over space, points equidistant from the origin have the same level of predicted interaction, ceteris paribus. For example, x, y boundary locations at equivalent distances from the origin are, from the perspective of a hypothetical interactor at the origin, equally attractive. When b varies spatially, however, this property no longer holds, since the equidistant points along the boundary now have different predicted interactions by virtue of their varying b values. We can, however, depict the transformation of Cartesian space that is indicated by a spatially varying b through a remapping of x and y that takes into account the alterations in relative positions resulting from the varying coefficient. This new map portrays “interaction space,” though the transformation will be distorted from the original Cartesian space. Indeed, the degree of contortion of the warped map is a function of the spatial biases in distance friction.

To generate a warped space map we determined the value of b along the x, y values of a boundary file using the estimated expansion equation (3). The absolute values of the boundary file coordinates were then raised to the power of the absolute values of b. The signs of the original x, y values were employed to reposition the b-modified (x', y') values to their original quadrant positions, and the x', y' coordinates were mapped in Cartesian space. Whereas the original x, y values expressed the boundary in Cartesian space, the new x', y' values reflect an interaction space which, when mapped, illustrates the
predicted interaction pattern in Cartesian space. In the following section the estimation of spatially varying distance-decay parameters is illustrated and the corresponding space warp maps are presented.

Data Analyses

Our empirical example uses migration data from State Economic Areas (SEAs) from 1965–70. Five origin SEAs, Atlanta, Detroit, Los Angeles, New York, and St. Louis, were selected to represent different regions of the United States. Separate origin-based models were estimated for migration flows out of these origins to the 77 largest SEAs in 1970. The dependent variable, \( I_p \), is the number of people over five years of age living in the origin SEA in 1965 and in one of the destination SEAs in 1970 (US Bureau of the Census 1972). The measure of mass, \( M_p \), is the over-five population living in the destination SEA in 1970. The measure of distance, \( d_p \), is the number of highway miles between each pair of cities (Rand McNally & Co. 1973). The \( x, y \) coordinates for each of the cities were digitized (US Geological Survey 1975) to generate the trend surface variables; the same procedure provided the 943 \( x, y \) locations in the boundary file. Both sets of \( x, y \) coordinates were then centered with respect to the mean of the destination cities’ coordinates.

The estimated models also included two measures aimed at controlling for spatial variation in economic opportunity at the destinations: the destination SEA’s 1969 mean family wage or salary income (\( W_d \)), and the destination SEA’s 1970 rate of civilian labor force unemployment (\( U_d \)). By augmenting equations (2) and (4) with these additional economic measures, the analyses reported below more closely conform to those undertaken in the standard migration literature (Greenwood 1975). Even so, the models are not meant to explain fully migration and its controlling factors, but to focus on assumptions regarding the nature of distance. Finally, following the suggestion of Fotheringham (1983, 1984), our analyses also include a “competing destinations” variable, \( A_p \), to control for the effect of the spatial structure of destinations on interaction. All three additional variables are logarithmically transformed.

Ordinary least squares estimates of the initial model—equation (2) plus \( \ln W_p, \ln U_p, \ln A_p \)—for the five origin cities were obtained by a backward elimination procedure terminated when all variables retained in the model were significant at the 95% level (Table 1). The initial model parameter estimates were consistent with expectations, though in general the economic measures played a minor role in the statistical explanation of migration flows.

We have opted here for expanding the distance-decay parameter into a second order polynomial, as shown in equation (3). This level of resolution provides for linear, quadratic, and interaction effects in both \( x \) and \( y \), and is a parsimonious solution for this illustration. The terminal model is equation (4) augmented by the additional economic and competing destination variables. The results were again obtained using a backward stepwise selection procedure (Table 2). Distance-decay parameter instability is indicated by significant spatial estimates of some of the parameters \( b_1 \) through \( b_5 \) for all cities except Los Angeles.

To obtain a portrait of the spatial instability of the friction of distance for migration over the 1965–70 time period, the significant \( b \) estimates (from Table 2) were replaced into their respective locations in expansion equation (3). The estimated expansion equations were then evaluated both for the 77 destinations’ \( x, y \) coordinates and for the \( x, y \) boundary coordinates. The former data were used to prepare contour maps of the friction of distance in the four cases in which this effect varied over space (Figs. 1a, 2a, 3a, 4a). The latter data were used to prepare space warp maps in the manner described above (Figs. 1b, 2b, 3b, 4b). For the purposes of this presentation we have chosen to render each of the warped space maps
roughly equal in size. However, the reader should be aware that inasmuch as the magnitudes of parameters vary from figure to figure, so too do the corresponding warped maps vary in size, with larger negative values implying a larger space.

The friction of distance map for migration out of Atlanta shows values of $b$ increasing towards the central portion of the country and attaining their highest effect in the northern tier states (Fig. 1a). Similar values of $b$ are found in the South-

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**TABLE 1**

OLS ESTIMATES OF MIGRATION MODEL

<table>
<thead>
<tr>
<th>Term</th>
<th>Atlanta</th>
<th>Detroit</th>
<th>Los Angeles</th>
<th>New York</th>
<th>St. Louis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>18.243</td>
<td>13.851</td>
<td>-0.949</td>
<td>18.887</td>
<td>-3.081</td>
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<tr>
<td>$\ln(M_i)$</td>
<td>1.086</td>
<td>0.994</td>
<td>0.948</td>
<td>1.037</td>
<td>0.913</td>
</tr>
<tr>
<td></td>
<td>(0.106)*</td>
<td>(0.010)</td>
<td>(0.066)</td>
<td>(0.105)</td>
<td>(0.108)</td>
</tr>
<tr>
<td>$\ln(d_i)$</td>
<td>-1.450</td>
<td>-0.719</td>
<td>-0.839</td>
<td>-0.864</td>
<td>-0.579</td>
</tr>
<tr>
<td></td>
<td>(0.149)</td>
<td>(0.137)</td>
<td>(0.092)</td>
<td>(0.145)</td>
<td>(0.149)</td>
</tr>
<tr>
<td>$\ln(A_i)$</td>
<td>-1.487</td>
<td>-1.353</td>
<td>-0.860</td>
<td>-1.676</td>
<td>-1.358</td>
</tr>
<tr>
<td></td>
<td>(0.213)</td>
<td>(0.254)</td>
<td>(0.136)</td>
<td>(0.378)</td>
<td>(0.188)</td>
</tr>
<tr>
<td>$\ln(U_i)$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>-0.583</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.230)</td>
</tr>
<tr>
<td>$\ln(W_i)$</td>
<td>—</td>
<td>—</td>
<td>1.306</td>
<td>—</td>
<td>1.887</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.517)</td>
<td></td>
<td>(0.868)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.709</td>
<td>0.646</td>
<td>0.916</td>
<td>0.688</td>
<td>0.736</td>
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<tr>
<td>Mean squared error</td>
<td>0.624</td>
<td>0.566</td>
<td>0.150</td>
<td>0.549</td>
<td>0.427</td>
</tr>
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*Standard errors reported in parentheses below the estimates. All estimates significant at the 95% level.

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**TABLE 2**

OLS ESTIMATES OF EXPANDED MODEL

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<th>Term</th>
<th>Atlanta</th>
<th>Detroit</th>
<th>Los Angeles</th>
<th>New York</th>
<th>St. Louis</th>
</tr>
</thead>
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<td>Constant</td>
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<td>-15.293</td>
<td>-15.253</td>
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<tr>
<td>$\ln(M_i)$</td>
<td>1.098</td>
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<td>0.948</td>
<td>0.765</td>
<td>0.744</td>
</tr>
<tr>
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<td>(0.091)*</td>
<td>(0.092)</td>
<td>(0.066)</td>
<td>(0.086)</td>
<td>(0.111)</td>
</tr>
<tr>
<td>$\ln(d_i)$</td>
<td>-1.490</td>
<td>-1.176</td>
<td>-0.839</td>
<td>-0.434</td>
<td>-1.088</td>
</tr>
<tr>
<td></td>
<td>(0.201)</td>
<td>(0.163)</td>
<td>(0.092)</td>
<td>(0.158)</td>
<td>(0.299)</td>
</tr>
<tr>
<td>$\ln(x_i d_i)$</td>
<td>0.016</td>
<td>—</td>
<td>—</td>
<td>0.037</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
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<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>$\ln(x_i y_i)$</td>
<td>-0.027</td>
<td>-0.029</td>
<td>—</td>
<td>-0.019</td>
<td>-0.019</td>
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<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
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<td>(0.007)</td>
<td>(0.008)</td>
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<tr>
<td>$\ln(x_i y_i)$</td>
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<td>—</td>
<td>0.004</td>
<td>0.003</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\ln(x_i y_i)$</td>
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<td>0.002</td>
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<td>0.002</td>
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<tr>
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<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\ln(y_i d_i)$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\ln(A_i)$</td>
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<td>-0.924</td>
<td>-0.860</td>
<td>-1.950</td>
<td>-1.768</td>
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<tr>
<td></td>
<td>(0.307)</td>
<td>(0.248)</td>
<td>(0.136)</td>
<td>(0.310)</td>
<td>(0.454)</td>
</tr>
<tr>
<td>$\ln(U_i)$</td>
<td>—</td>
<td>—</td>
<td>1.306</td>
<td>4.058</td>
<td>4.201</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(0.517)</td>
<td>(0.729)</td>
<td>(0.956)</td>
</tr>
<tr>
<td>$\ln(W_i)$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.814</td>
<td>0.729</td>
<td>0.916</td>
<td>0.883</td>
<td>0.775</td>
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<tr>
<td>Mean squared error</td>
<td>0.417</td>
<td>0.446</td>
<td>0.150</td>
<td>0.222</td>
<td>0.380</td>
</tr>
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*Standard errors reported in parentheses below the estimates. All estimates significant at the 95% level.
Figure 1. Migration from Atlanta. (a) Distance decay for migration flows. (b) Corresponding warped space map.
Figure 2. Migration from Detroit. (a) Distance decay for migration flows. (b) Corresponding warped space map.
Figure 3. Migration from New York. (a) Distance decay for migration flows. (b) Corresponding warped space map.
west and along the eastern seaboard. The transformation of Cartesian space portrays a greater effect of distance and consequently less migration to the northern interior of the country (Fig. 1b). The eastern seaboard, Southeast, and Southwest, on the other hand, are relatively closer to Atlanta, indicating higher migration propensities to those regions, ceteris paribus. The picture emerging from this map is one of compressed space throughout the Sunbelt and East Coast, and extended distances elsewhere, particularly in the Northwest.

Detroit shows a simple Sunbelt-Snowbelt pattern, with lower b values encountered in directions south and west of the origin (Fig. 2a). In fact, the contours show that movement out of Detroit generally involves less friction the further away the destination. The same is true of predicted migration (Fig. 2b). Locations north of Detroit are stretched compared to conventional maps, while distances to the south and west are reduced. Los Angeles, for example, is relatively closer to Detroit in warped space than it is in actual space.

New York’s parameter values are lower than those found in the other maps, indicating less friction of distance out of this origin (Fig. 3a). The pattern shows extremely low values along the East Coast; larger values prevail in the West, though distance decay is reduced again in the Southwest. The considerable differences in parameter values for the New York model are readily apparent in the dramatic transformation of space in the warped map (Fig. 3b). Physical distance along the southeastern coast has been obliterated by low and decreasing distance-decay parameters in the eastern portion of the country. As a result, the entire eastern seaboard is represented as a series of overlapping lines, indicating that distance plays little role in deterring migration in this region. The compression effect is similar in the Southwest, resulting in a distorted space along the West Coast. In the northern portion of the country, on the other hand, distances are magnified by higher parameter values.

For St. Louis, the contour map indicates a bias towards the eastern seaboard and the South and Southwest (Fig. 4a). Higher parameter values are encountered as one moves from the upper Midwest to the northwestern states. This pattern of distance decay is illustrated in the stretched boundary to the northwest of the origin city, while the boundaries of the Southwest and South are considerably closer to St. Louis than they are in actual distance (Fig. 4b).

These results are clearly consistent with the well-documented migration patterns from northern US regions to those in the South and West during the 1965–70 period (see Berry and Dahman 1980; Roseman 1977; Sternlieb et al. 1982). In all cases, this consistency is revealed by increasingly negative parameter values in the northern regions of the country and lower values in the South. The maps also show a bias for migration towards the Southwest. The “why” of the parameter variation identified here remains problematic but has several likely sources. One is the variability of information about potential destinations. Distance decay in migration is often partially attributed to the effect that distance has on information about opportunities in potential destinations. The variability identified here could conceivably reveal differences in information flows at the regional level. Other possible factors include socio-cultural preferences for specific regions or disparities in the perception of distances due to previous experience in certain regions.

Conclusion

The model-to-model variation of distance-decay parameters has attracted considerable interest on the part of geographers, but with few exceptions (Brown and Jones 1985; Casetti and Can 1986; Eldridge 1987) most have been content to assume within-model parameter stability. This research suggests that suspicion of such assumptions is warranted, since the effect of distance was found to vary in four of the five cities examined. Here the spatially uneven effects of distance were portrayed as warped space, a term which refers to the cartographic representation
Figure 4. Migration from St. Louis. (a) Distance decay for migration flows. (b) Corresponding warped space map.
of unequal distance effects through transformations of Cartesian space.

Our findings prompt us to make some general comments on the nature of the modeling process and on the conceptions of space that inform it. One view of modeling, perhaps not as outdated as some might believe, tends to emphasize the identification of consistent empirical regularities that might ultimately lead to the formulation of universal spatial "laws." The search for a single distance-decay parameter, itself reliant upon the objectification of space, is an extreme example of this. Even the assumption of within-model parameter stability is consistent with attempts at universal spatial explanation. Indeed, any approach towards modeling that does not explicitly question the contextual specificity of parameters is derivative of the law-seeking version of scientific investigation.

An alternative view is provided by the expansion method, which rejects the notion that models should perform in a similar fashion across varied contexts. Parameter instability is recast from dilemma—that is, a problem of misspecification—to research focus. Model variability is viewed as an inevitable result of the complexity of phenomena, as indicative of context, and as a clue toward richer and more realistic theory construction (Jones 1992).

The expansion method led us to reject the assumption that the effect of distance is everywhere constant. This assumption underlies not only spatial interaction models, but much of location theory in general, and one might even suggest that by imposing a homogeneity of distance, such endeavors have paradoxically stripped away aspects of spatial difference they aimed to explain. In the case under consideration here we can note that there is no reason to treat the distance-decay effect as a universal entity devoid of context.

**Literature Cited**


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